

Matched-Filter Detection of Mode-Locked Laser Signals

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The passive Fabry-Perot cavity is shown to be a good practical approach to the match-filter optimization for the sensitive detection of mode-locked laser signals. Doppler measurements of relative motion over a wide range of velocities are possible simply by measuring the cavity length for a peak output.

I. Introduction

Multitone lasers of the mode-locked¹ and cavity-dumped² types emit their energy in short pulses. Sensitive detection of these emissions for point-to-point communications or echo-ranging systems can be accomplished by using appropriate filtering at the optical frequencies for detection and radio-frequency amplification. In the present work we describe our approach to matched-filtering for these signals.

In the literature related prior studies of laser detection include scanning Fabry-Perot cavities using a single pass-band of the passive cavity to analyze laser radiation³, spatial filtering techniques⁴, and various laser heterodyne techniques.⁵⁻⁷

Our optical receiver for mode-locked gas laser signals consists of a passive laser cavity controlled in length and a photodetector with its associated electronics. The length of the passive Fabry-Perot cavity is chosen approximately equal to the cavity length of the transmitting laser but with provision for fine fractional wavelength control of its length. In addition to the selective filtering characteristics of the passive cavity (passbands of unity transmission matching the frequencies of the multimode laser), a readout of the vernier length control, peaking the output, provides for an extremely wide range of velocity measurements with either an active or passive vehicle moving relative to the receiver.

II. Passive Cavity: Matched-Filter for Mode-Locked Laser Radiation.

Consider the passive cavity as shown in Fig. 1 where h_0 is the cavity length, M_1 is the fixed cavity mirror, and M_2 is the movable cavity mirror. The amplitude transmission function $T(\omega)$ may be shown to be⁸

$$T(\omega) = L \exp(i\psi(\omega)) / \left[1 + P_2 \sin^2\left(\frac{\omega h_0}{c}\right) \right]^{1/2} \quad (1)$$

where

$$P_2 = [4r_1 r_2 \exp(-2\alpha h_0)] / [1 - r_1 r_2 \exp(-2\alpha h_0)]^2 \quad (2)$$

and

$$\psi(\omega) = -(\omega/c)h_0 - \tan^{-1} \left\{ \frac{r_1 r_2 \sin[2\omega(h_0/c)]}{1 - r_1 r_2 \cos[2\omega(h_0/c)]} \right\} \quad (3)$$

and

$$L = [t_1 t_2 \exp(-\alpha h_0)] / [1 - r_1 r_2 \exp(-2\alpha h_0)] \quad (4)$$

t_1 and t_2 are the transmission functions for mirror 1 and mirror 2 respectively, r_1 (mirror 1 right-side incidence) and r_2 (mirror 2 left-side incidence) are the reflection coefficients, α is the cavity loss per unit length, and c is the speed of light.

To study the passive cavity as a multitone filter we use the matched-filter criterion resulting from the optimization of the SNR. So for a linear system with impulse response function $H_1(t)$ the total time varying output is $G_1(t) = H_1(t) \otimes [F_1(t) + N_1(t)]$ where $N_1(t)$ is the additive input signal and \otimes indicates convolution. The matched-filter criterion specifies that the amplitude transmission function $H(\omega)_m = AF^*(\omega)/S_n(\omega)$ where $H(\omega)_m$ and $F(\omega)$ are the Fourier transform of $H_1(t)$ and $F_1(t)$ respectively, $S_n(\omega)$ is the power spectral density of the additive input noise, the asterisk denotes the complex conjugate, and A is any nonzero complex constant.⁹ The subscript 1 will be used to denote time varying signals. Hence we see that the signal for which the passive cavity is a matched-filter is given by $F(\omega) = S_n(\omega) T^*(\omega)/A^*$. Since the signals we shall be considering have a finite number of frequency peaks, we shall approximate $T(\omega)$ expanding around the zeroes, ω_p , of $\sin^2(\omega h_0/c)$ for a finite number of peaks around ω_0 so that from Eq. (1) dropping $\psi(\omega)$ we have

$$T(\omega)_A = L / \left[1 + \left(\frac{2\Delta\omega}{\Delta\omega_p} \right)^2 \right]^{1/2} \quad (5)$$

where $\Delta\omega = \omega - \omega_p = \omega - (\omega_0 + p\omega_{co})$, $\omega_{co} = \pi c/h_0$ and $\Delta\omega_p = 2c/[h_0(P_2)^{1/2}]$. We note that $\Delta\omega_p$ is the

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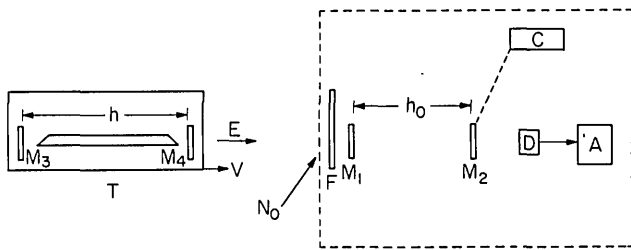


Fig. 1. Passive cavity receiver: T laser transmitter; M_3, M_4 laser mirrors; h laser cavity length; E signal; V velocity of laser relative to receiver; N_0 noise; F coarse bandpass filter; M_1, M_2 passive cavity mirrors; h_0 passive cavity length; D detector; A detector electronics; C mirror control.

full width at half power of each Lorentzian lineshape function generated by Eq. (5). Generally we shall assume white noise so that $S_n(\omega) = N_0$ is uniform over the frequencies of interest. Similarly we shall assume t_1, t_2, r_1, r_2 are constant over the frequencies of interest. Thus from Eq. (5) we see that the signal for which the passive cavity is a matched-filter is

$$F(\omega) = [S_n(\omega)L]/A^* \sum_{p=-N}^N 1/\left[1 + \left(\frac{2\Delta\omega}{\Delta\omega_p}\right)^2\right]^{1/2}. \quad (6)$$

Now for convenience we shall assume the idealized electric field amplitude at the output of an unmodulated mode-locked laser with $2N + 1$ modes is given by

$$E_1(t) = \sum_{p=-N}^N \exp[i(\omega_0 + p\omega_c)t] \\ = \exp(i\omega_0 t) \left\{ \sin\left[(2N + 1)\frac{\omega_c t}{2}\right] / \sin\left(\frac{\omega_c t}{2}\right) \right\} \quad (7)$$

where $\omega_0 = n\pi c/h$ is the center frequency of the laser, n is some large integer, $\omega_c = \pi c/h$ is the free spectral range, and h is the effective cavity length. Then the Fourier transform of Eq. (7) is

$$E(\omega) = 2\pi \sum_{p=-N}^N \delta[\omega - (\omega_0 + p\omega_c)] \quad (8)$$

where $\delta(\omega)$ is the Dirac delta function. Thus we see that $T(\omega)_A$ is a comb-filter for $E(\omega)$ for $h = h_0$, i.e., $T(\omega)_A$ has passbands of unity transmission matching the frequencies of the multimode laser.

For an actual laser signal, writing $E(\omega)$ for a multitone laser with finite linewidth will yield an expression as an alternative to the monochromatic idealization of Eq. (8). We note that the spacing between the tones of a mode-locked laser are determined by free spectral range, $\omega_c = \pi c/h$, of the laser cavity. It is our contention that the passive cavity transmission function will control the lineshape of the laser output if the gain α and the dispersion in the cavity are independent of frequency around a resonant peak. One may further consider this observation by noting that for He-Ne the width of the Lorentzian shaped hole that is burned into the Doppler broadened gain profile at saturation is much broader than the mode-width of the laser cavity. Thus for $\alpha < 0$

the frequency variation of P , Eq. (2), and L , Eq. (4), are negligible around the resonance.

In the literature related prior studies of the laser lineshape include lumped element LGC circuit models.^{10,11} Freed and Haus¹² used the solution of the nonlinear Van der Pol oscillator equation to describe the spectrum of the laser output. The basic result of using a lumped circuit model is that the lineshape function is Lorentzian.¹³ The interesting point is that if one started with the transmission function for a cavity or a transmission line, the lineshape would be controlled by equations similar to Eq. (1).

Note that both the lumped circuit and passive cavity approaches to the laser spectrum assume that the random cavity mirror vibrations and effective cavity length fluctuations are negligible. If one dropped these assumptions the lineshape could be considered Gaussian or some other lineshape function.

So as the gain curve saturates, the right-half-plane poles of Eq. (1), for $s = i\omega$, will migrate to the $i\omega$ axis. We select the value of α which is an amount ϵ from the saturated pole so that Eq. (2) becomes

$$P_1 = [4 \exp(-2\epsilon h_0)]/[1 - [\exp(-2\epsilon h_0)]^2]. \quad (9)$$

Then we may write the spectrum of the laser signal with finite linewidths as

$$E(\omega) = t_1 t_2 / [2(r_1 r_2)^{1/2}] \sqrt{P_1} \{1/[1 + P_1 \sin^2 \omega(h_0/c)]^{1/2}\}. \quad (10)$$

We note that a very good approximation to Eq. (10) may be obtained by expanding around the zeroes ω_p of $\sin^2 \omega(h_0/c)$ for $2N + 1$ tones analogous to the approximation of Eq. (5) so that the alternative to Eq. (8) becomes

$$E(\omega) = (t_1 t_2) / [2(r_1 r_2)^{1/2}] \sqrt{P_1} \sum_{p=-N}^N 1/\left[1 + \left(\frac{2\Delta\omega}{\Delta\omega_1}\right)^2\right]^{1/2} \quad (11)$$

where $\Delta\omega = \omega - \omega_p = \omega - (\omega_0 + p\omega_c)$, $\Delta\omega_1 = 2c/(h\sqrt{P_1})$. Thus comparing Eq. (6) with Eq. (11) indicates that for $\Delta\omega_p = \Delta\omega_1$ and $h = h_0$ the passive cavity of Eq. (1) is a matched-filter for the multitone signal of Eq. (10).

III. Signal-to-Noise Ratio

We compute the predetection time varying SNR at the output of the passive cavity as

$$\text{SNR}(t) = |T_1 \otimes E_1|^2 / R_{nn}(0) \quad (12)$$

where E_1 is the inverse Fourier transform of Eq. (10) and T_1 is the inverse Fourier transform of Eq. (1) and $R_{nn}(0)$ is the auto correlation function evaluated at zero, which is equivalent to the mean-squared value of the additive noise at the output of the passive cavity given by

$$R_{nn}(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_n(\omega) |T(\omega)|^2 d\omega. \quad (13)$$

The numerator of Eq. (12) may be expressed by the inverse Fourier transform as

$$|T_1 \otimes E_1|^2 = \left| \frac{1}{2\pi} \int_{-\infty}^{\infty} T(\omega) E(\omega) \exp(i\omega t) d\omega \right|^2 \quad (14)$$

and substituting Eq. (1) and Eq. (10) we may write the modulus-squared value of the signal portion of the output of a passive cavity as

$$|T_1 \otimes E_1|^2 = \left| \frac{L t_1 t_2}{2(r_1 r_2)^{1/2}} \int_{-\infty}^{\infty} \exp(i\omega t) d\omega \right. \\ \left. \times \frac{1}{[1 + P_1 \sin^2 \omega(h_0/c)]^{1/2} [1 + P_2 \sin^2 \omega(h_0/c)]^{1/2}} \right|^2 \quad (15)$$

Since the signal has only $q = 2N + 1$ modes we may write Eq. (15), with the substitutions $\omega = \omega_0 - (c/h_0)x$, $t = h_0/c$, $\omega_0 = (n\pi c)/h_0$, where n is some larger interger, $\exp(-ix) = \cos x - i \sin x$ and noting that the sine integral over symmetric limits vanishes, as

$$|T_1 \otimes E_1|^2 = (q/2\pi)^2 L^2 \left[\frac{t_1 t_2}{2(r_1 r_2)^{1/2}} \right]^2 (2c/h_0)^2 K^2(m) \quad (16)$$

where for $1/P_2 > 1/P_1 > 0$, $K(m)$ is the complete elliptic integral of the second kind given by

$$K(m) = \int_0^{\pi/2} [(d\theta)/(1 - m^2 \sin^2 \theta)^{1/2}]$$

and

$$m = [(1/P_2) - (1/P_1)]^{1/2} / (1/P_2)^{1/2}.$$

The denominator of Eq. (12) may be expressed from Eq. (1) and Eq. (13) as

$$R_{hn}(0) = \frac{L^2}{2\pi} \int_{q[(\pi c)/h_0]} \frac{S_n(\omega) d\omega}{1 + P_2 \sin^2[\omega(h_0/c)]} \\ = \frac{L^2 N_0}{2\pi} q \frac{\pi c}{h_0} \frac{1}{(1 + P_2)^{1/2}} \quad (17)$$

where, as in the integration above, q is the number of free spectral ranges over which the integration is taken. Thus the peak SNR for the passive cavity taking the ratio of Eq. (16) to Eq. (17) is given by

$$\text{SNR}_p = 4(q/2\pi)[t_1 t_2 / 2(r_1 r_2)^{1/2}]^2 (c/h_0) (1 + P_2)^{1/2} K^2(m) \\ / (\pi N_0). \quad (18)$$

In a similar fashion, expressing $E \otimes E^*$, and $R_{hn}(0)$ for the matched-filter to Eq. (10) and using the same assumptions as above, the peak SNR for the matched-filter is

$$\text{SNR}_i = 4(q/2\pi)[(t_1 t_2) / [2(r_1 r_2)^{1/2}]]^2 (c/h_0) [\tan^{-1}(\sqrt{P_1})]^2 \\ \times (1 + P_1)^{1/2} / (\pi N_0). \quad (19)$$

Thus the departure of the passive cavity from the matched-filter is from Eqs. (18) and (19)

$$\text{SNR}_p / \text{SNR}_i = (1 + P_2)^{1/2} / (1 + P_1)^{1/2} [K^2(m) / [\tan^{-1}(\sqrt{P_1})]^2]. \quad (20)$$

Since $P_1 \gg 1$, $\Delta\omega_p = 2c/h_0 \sqrt{P_2}$, and $\Delta\omega_1 = 2c/(h_0 \sqrt{P_1})$ then

$$\text{SNR}_p / \text{SNR}_i = (2/\pi)^2 (\Delta\omega_1 / \Delta\omega_p) K^2(m). \quad (21)$$

Hence the SNR of the passive cavity approaches that of the matched-filter when $\Delta\omega_l \cong \Delta\omega_p$ as may be seen in the plot of Eq. (21) in Fig. 2.

The Fabry-Perot cavity is probably as close a physical realization to a matched-filter for the multi-toned laser as can be attained in a passive system. Even so, gain narrowing invariably results in $\Delta\omega_l < \Delta\omega_p$, thereby limiting the observed improvement in SNR from its optimal value.¹⁴ For high gain lasers with cavities of low finesse, the receiver can be made closer to ideal, while greater departures are to be expected in the case of low gain. We note, too, that larger bandwidths, $\Delta\omega_p$, are called for with information modulated lasers and cavity-dumped lasers where mode-locking may not have been employed.

To obtain the predetection SNR improvement with use of the passive cavity we note the SNR without the cavity is given by the ratio of the modulus-squared value of Eq. (7) to Eq. (17) with $P_2 = 0$. Thus the SNR improvement with use of the passive cavity with respect to no cavity is given by Eqs. (1), (7), (12), (14), and (17) as

$$\text{SNR}_{\text{with}} / \text{SNR}_{\text{without}} = (1 + P_2)^{1/2}. \quad (22)$$

Since the signal portion of the time varying output of the passive cavity for a mode-locked laser input is given by the real part of $T_1 \otimes E_1$, from Eq. (1) and Eq. (8) dropping $\psi(\omega)$ we have

$$G_1(t) = \sum_{p=-N}^N [\cos(a - b)] / (1 + P_2 \sin^2 b)^{1/2} \quad (23)$$

where $a = \{\omega_0 + [\pi c/h]\}t$ and $b = (h_0/c) \{\omega_0 + [(p\pi c)/h]\}$, then we see the peak SNR occurring for $h = h_0$. The SNR as a function of time and a function of relative cavity length h_0/h is illustrated in Fig. 3. The number of modes oscillating is 9 with a peak SNR of 49 and a period of 8 nsec. The increment of relative cavity length is $\Delta h_0/h = 0.0001$.

IV. Rise-Time of the Passive Cavity

In consideration of the rise-time, τ , of the Fabry-Perot resonant cavity we start with the Laplace transform representation of the amplitude transmission function given by setting $s = i\omega$ in Eq. (1) we obtain

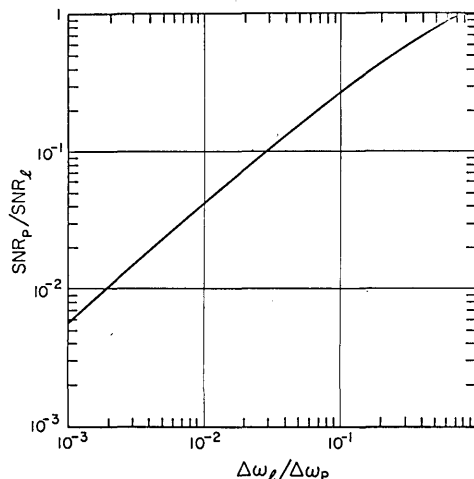


Fig. 2. Departure from matched-filter vs relative line widths.

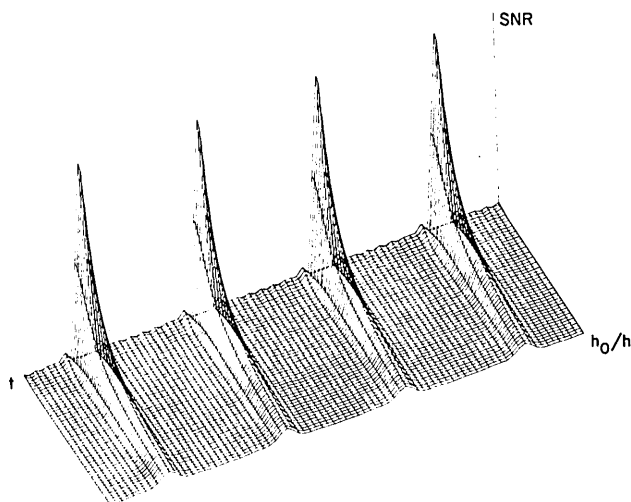


Fig. 3. Signal-to-noise as a function of time and relative cavity length.

$$T(s) = \exp(-as)T'(s) = [T_\alpha \exp(-as)]/[1 - R_\alpha \exp(-2as)] \quad (24)$$

where $T_\alpha = t_1 t_2 \exp(-\alpha h_0)$, $R_\alpha = r_1 r_2 \exp(-2\alpha h_0)$, $a = (h_0/c)$. If we expand $T'(s)$ in partial fractions and group the complex conjugate poles we obtain

$$T'(s) = \frac{T_\alpha}{a} \sum_{m=0}^{\infty} (s - \sigma)/[(s - \sigma)^2 + \omega_m^2] \quad (25)$$

where $\alpha = 1/(2a) \ln R_\alpha$ and $\omega_m = m\pi c/h_0$. Now the Laplace transform of the real part of Eq. (7) becomes

$$E(s) = L[\text{Real } E_1(t)] = \sum_{p=-N}^N [s/(s^2 + \omega_p^2)] \quad (26)$$

So the signal output of the passive cavity for the idealized mode-locked laser input is given by

$$G(s) = \exp(-as)G'(s) = \exp(-as)E(s)T'(s) \quad (27)$$

where we note that $\exp(-as)$ for a non-negative real constant corresponds to a time shift in the time domain. The inverse Laplace transform of a general term of $G'(s)$ is given by

$$L^{-1} \left[\frac{s}{s^2 + \omega_p^2} \cdot \frac{(s - \sigma)}{(s - \sigma)^2 + \omega_m^2} \right] = \left[\frac{1 + \sigma^2}{(\sigma^2 + \omega_m^2 - \omega_p^2)^2 + 4\sigma^2\omega_p^2} \right]^{1/2} \times [\omega_p \sin(\omega_p t + \psi_1) + \omega_m \sin(\omega_m t + \psi_2)] \quad (28)$$

Thus for $\omega_p = \omega_m \gg \sigma$, $\sigma \gg 1$ the fraction of the maximum steady state output which is obtained in τ seconds is given by Eq. (27) and Eq. (28) as

$$f = 1 - \exp[\sigma(\tau - a)]u(\tau - a) = (1 - R_\alpha^{(\tau-a)/2a})u(\tau - a) \quad (29)$$

where

$$u(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0 \end{cases}$$

The $\omega_p \neq \omega_m$ terms are neglected in that the coefficients of the sine terms of Eq. (28) are on the order of $\ln R_\alpha$ which are negligible for $R_\alpha \approx 1$. Equation (29) is plotted in Fig. 4 as a function of time and number of pulse train bounces parameterized by $R_\alpha = r_1 r_2 \exp(-2\alpha h)$.

Thus the rise-time of the passive cavity is given by

$$\tau = -1/\sigma = -2a[1/(\ln R_\alpha)] \approx 2(h_0/c) R_\alpha/(1 - R_\alpha) \quad (30)$$

and from Eq. (2)

$$\tau = (h_0/c) \sqrt{P/R_\alpha} = (2\sqrt{R_\alpha})/(\Delta\omega_p) = \sqrt{R_\alpha}/(\pi\Delta\nu_p) \quad (31)$$

and thus the rise-time-bandwidth product is

$$\tau\Delta\nu_p = \sqrt{R_\alpha}/\pi. \quad (32)$$

Where $R_\alpha = 0.991$, $\tau\Delta\nu_p = 0.3175$.

V. Doppler Measurements with the Passive Cavity

Suppose the mode-locked laser is moving toward our receiver with a velocity v as indicated in Fig. 1. For TEM waves,¹⁵ an emitted frequency ω' will be observed up-shifted to ω given by $\omega = \gamma(1 + v/c)\omega'$ in which $\gamma = [1 - (v/c)^2]^{1/2}$. Assuming normal incidence, by Eq. (7) the input signal, i.e., the Doppler shifted electric field, is readily expressed as:

$$E_1(t) = \{\sin[(2N + 1)\omega_c\gamma(1 + v/c)t/2]\}/\{\sin[\omega_c\gamma \times (1 + v/c)t/2]\} \cdot \exp[i\omega_0\gamma(1 + v/c)t] \quad (33)$$

Thus, in the case where there is relative motion, optimal detection of the mode-locked laser signal requires a receiver with a cavity length, h_0 , given by

$$h_0 = \pi c/\{\omega_c\gamma[1 + (v/c)]\} = h/\{\gamma[1 + (v/c)]\} \quad (34)$$

Similarly if the mode-locked laser and the passive cavity were on a common platform, the echo from a vehicle moving toward this platform with velocity v would be shifted to $\omega = (1 + 2v/c)\omega'$, where we have set $\gamma = 1$. So by vernier adjustments (PZT driven mirror) of the passive cavity we can read a large range of approach velocities, with a resolution independent of the velocity v , i.e. $\delta h/h \approx \delta v/c$ for $v/c \ll 1$. Thus with $\delta h/h = 3.3 \times 10^{-8}$ we find a resolution of $\delta v \approx 10$ m/c.

VI. Experiment

In this experiment the SNR improvement by pre-detection filtering of the optical input by a passive Fabry-Perot resonant cavity was measured as a function of several parameters: relative cavity lengths and passive cavity finesse (Fig. 1). Since the passive cavity is a good approximation to the matched-filter for mode-locked lasers only when the cavity lengths are matched, the detector output was monitored for various relative cavity lengths. The mode-locked laser signals were obtained from a self-mode-locking He-Ne laser operating at $0.633 \mu\text{m}$ of length 1.2 m with an average power output of 2 mW. The change in length of the laser h was pro-

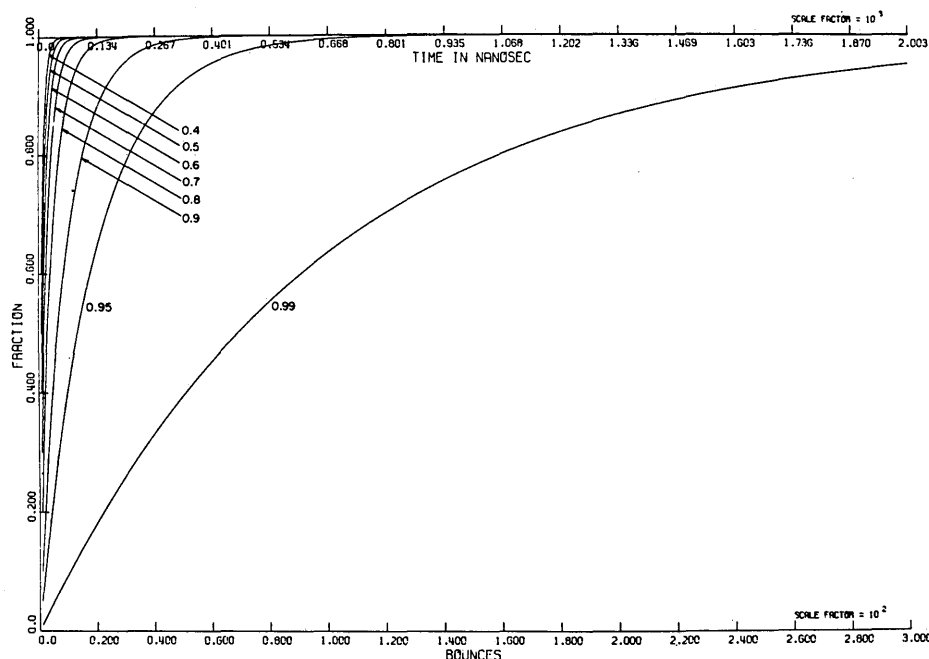


Fig. 4. Fraction of the maximum output vs time in nanoseconds and vs the number of complete bounces M , parameterized for various $R_\alpha = r_1 r_2 \exp(-2\alpha h)$.

vided by the motion of mirror M_3 on Teflon runners while the fine fractional wavelength control ($1 \mu\text{m}$ full scale) of the passive cavity length h_o was provided by a PZT driven mirror M_2 . The passive cavity was fabricated with the mirrors on Invar rods to reduce thermal variations while the entire experiment was performed in a controlled acoustical environment that provided isolation (~ 70 dB) from external turbulence and mechanical fluctuations. The passive cavity was scanned at a 1-Hz rate to eliminate the remaining fluctuations. Thus continuous monitoring of the cavity output, while the length of the laser cavity was changed, allowed us to match exactly the cavity lengths by peaking the output. Note that all the modes of the laser are transmitted simultaneously in the matched condition. The power output was observed to go as approximately $(2N + 1)^2$, i.e., as the square of the number of modes oscillating.

To measure the SNR improvement, white noise N_o from an ac driven tungsten lamp at 3200 K was introduced axially into the system. The power from the noise source, passed through a coarse band pass filter, F , at the input of the passive cavity, was 1.5 W in a 100-Å band around $0.633 \mu\text{m}$. The thermal noise of the TIXL55 avalanche diode detector in a 500-MHz range was far above the shot noise value $(2eI_o\Delta f)^{1/2}$ for operating currents of $0.1 \mu\text{A}$; so, throughout this discussion, we shall be considering the signal-to-unwanted signal ratio. Mirror M_1 is a standard Spectra-Physics flat laser reflector with reflectivity 0.991. Mirror M_2 is a standard Spectra-Physics Laser reflector with radius of curvature 2 m and reflectivity 0.991. Both reflectivities were chosen to maximize the ideal finesse $F \equiv \pi\sqrt{P}/2$ by minimizing loss. In all cases the measured finesse (245 max) was lower than the ideal (346) due to mirror surface roughness and scattering from occlusions

in the mirror multilayer. This factor as well as the finesse was measured by means of a Spectra-Physics Model 119 single mode laser which provided the delta in frequency required to study the spectral response of the cavity.

With the cavity set for the largest Fresnel number (50), i.e., end apertures were limited by the mirror dimensions, the signal and noise were measured with M_1 and M_2 aligned to maximize the passive cavity finesse. The finesse was also measured and found to be 245. The mirrors of the passive cavity were then removed and the signal and noise were measured again. The ratio of SNR_{with} to $\text{SNR}_{\text{without}}$ was 156, and from Eq. (22) we see that the theoretical improvement using the experimentally determined finesse is $2(245)/\pi = 156$. The experiment was repeated for various values of finesse and the results are summarized in Fig. 5 along with a plot of Eq.

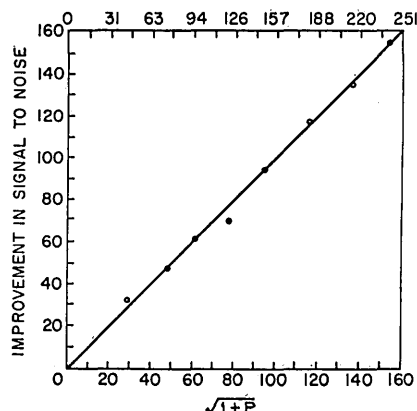


Fig. 5. Signal-to-noise improvement with passive cavity.

(22). All measurements were made using density filters calibrated at $0.633 \mu\text{m}$ to avoid nonlinearities in the detector electronics.

VII. Summary

In this work we have shown that the passive Fabry-Perot resonant cavity which is equal in length to the laser cavity is probably as close a physical realization to a matched-filter for multitoned mode-locked gas laser signals as can be attained in a passive system. For the passive cavity in contrast to no cavity the SNR improves by the factor $(P_2 + 1)^{1/2}$ which is typically 100–200 for a cavity of good finesse. Also, the peak value of the temporally varying SNR improves as $(2N + 1)^2$, i.e., as the peak power of the mode-locked laser.

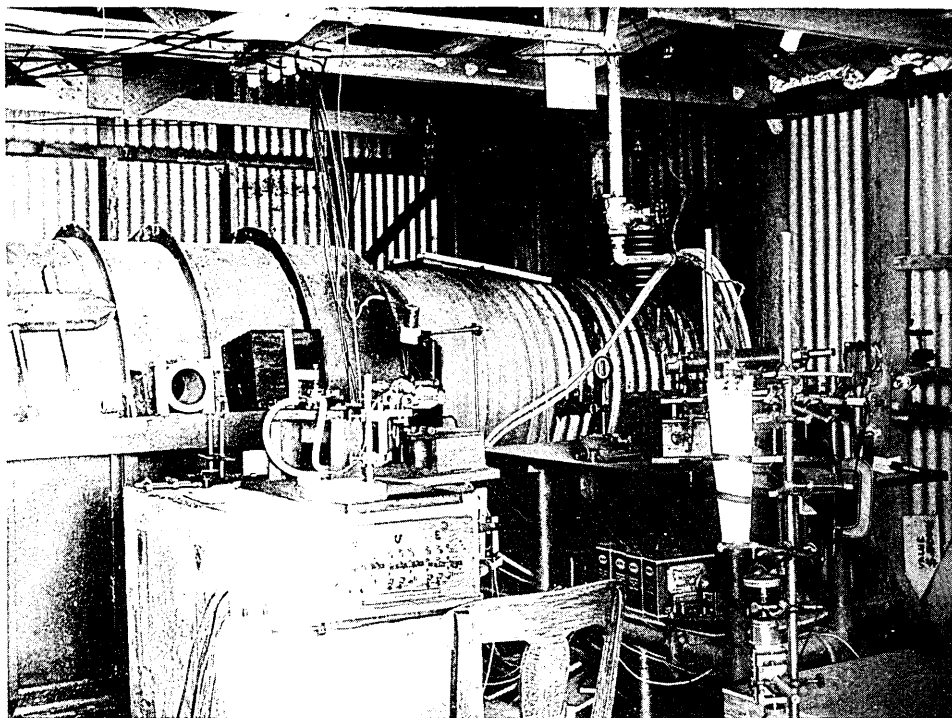
An alternative approach to the mode-locked laser lineshape is presented in Eq. (10) along with the departure of the passive cavity from the matched-filter for $\Delta\omega_1 < \Delta\omega_p$ and $h \neq h_0$. We obtain an expression for the rise-time which is also a function of $\Delta\omega_p$ and for cavities of high finesse independent of the number of modes detected.

Further improvement of the SNR above that obtained with the passive cavity may be accomplished with subsequent processing of the detector output, such as boxcar-integration of time-sampled displays. In applications of the receiver to information modulated multitone lasers, the effective bandwidth of the passive Fabry-Perot can be controlled by appropriate choices of r_1 and r_2 in Eq. (2).

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Interior of the shed at the Irvine Ranch housing the rotating mirror, telescope, etc. used in the experiment to measure the velocity of light in vacuum. (From the Michelson Museum collection)